Syllabus

1 Logistics

- Lectures: Mondays & Wednesdays, 11:30am – 1:00pm, McCullough Building 126.
  Note the first two weeks of lectures are online, Zoom link available through Canvas.
- Primary Instructor: Tselil Schramm (tselil@stanford.edu). Office Hours W 2-3:30 pm and by appointment, in Sequoia Hall 132.
- TAs: Marius Tirlea (Office Hours TBA) and Chenyang Zhong (Office Hours Tu 9-10:30 am).
- Course website: www.tselilschramm.org/random-processes/stats221-winter22.html

To contact the course staff please use email. Please be sure to include "STATS 221" in the subject line.

Prerequisites. MATH 115, "Functions of a real variable" (or equivalent), and STATS 217, "Introduction to Stochastic Processes" (or equivalent).

2 Overview & Learning Goals

Building on undergraduate probability (STATS 116), and on previous exposure to Markov chains (STATS 217), we'll cover modern topics in the study of random processes on graphs and lattices, in a non-measure-theoretic way. Specifically:

- The connection between reversible Markov chains, electrical networks and flows.
- The study of random graphs, uniform spanning trees, Percolation and self-avoiding walks.
- Dynamics of interacting particles: the contact process, voter model and the exclusion process.
- Gibbs states, in particular the Ising, Potts and Random-Cluster models.

We'll see the power of simple random models to illuminate real-world phenomena arising in physical systems, epidemiology, politics, population dynamics, and more. By focusing on discrete objects, we will see complete proofs without requiring measure theory or functional analysis. In contrast with STATS 217, many of the topics we cover are at the forefront of research in discrete probability (and over the course of the term, we will state a few well known open problems and tantalizing conjectures).

Course Description (as it appears in the course catalogue): Covering modern topics in the study of random processes on graphs and lattices. Specifically, a subset of: Random walks, electrical networks and flows. Uniform spanning trees. Percolation and self-avoiding walks. Contact process, voter model and the exclusion process. Ising, Potts, and Random-Cluster model. Random graphs.

Learning Goals.

1. Use the behavior of simple, discrete random models to explain real-world phenomena arising in physical systems, epidemiology, politics, population dynamics, and more.
2. Become conversant in topics at the forefront of research in discrete probability. Be able to engage with probability seminar talks in this domain.

3 Course Structure

Topics. We will cover the following topics, roughly partitioned into the following units:

1. Random walks on graphs: connections between random walks, Markov chains, electrical networks, and flows.
2. Uniform Spanning Trees: negative association, sampling algorithms, uniform forests.
3. Percolation: phase transitions, self-avoiding walks.
4. Association and concentration: conditions for stochastic dominance, positive association, negative association in product measures, concentration for martingales, influence and thresholds.
5. Interacting particle systems: the contact model, and if time allows, the voter and exclusion models.
6. Gibbs measures: the Ising, Potts, and random cluster models for physical states.

Schedule. Below is a preliminary schedule, including the textbook chapters relevant to each lecture.

Date	Topic	Grimmet
01/03	Random walks on graphs	1.1, 1.2, 1.6
01/05	Electrical networks and flows	1.3, 1.4
01/10	Recurrence and resistance	1.4, 1.5
01/12	Uniform spanning trees	2.1, 2.2
01/17	NO CLASS, Martin Luther King Jr. Day	-
01/19	Sampling uniform spanning trees	2.2
01/24	Percolation & phase transitions	3.1, 3.4
01/26	Self-avoiding walks	3.1, 3.2
01/31	Conditions for stochastic dominance & positive association	4.1, 4.2
02/02	Conditions for negative association, concentration of martingales	4.3, 4.4
02/07	Influence and thresholds	4.5, 4.7
02/09	Contact model	6.1, 6.2
02/14	Criticality for the contact model	6.3, 6.4
02/16	Criticality continued, & contact model on trees	6.4, 6.5
02/21	NO CLASS, President's Day	-
02/23	Ising and Potts models	7.1, 7.3
02/28	Ising and Potts models, continued	7.2
03/02	Random cluster model	8.1, 8.2
03/07 & 03/09	Student presentations	-

4 Materials & Resources

Textbook and other resources.

- Our course text: "Probability on Graphs" by Geoffrey Grimmet. You may use the free preprint which is available online. In addition, a number of physical copies of the text are on reserve for our course at the library.
- Additional recommended readings will be listed on a per-topic basis on the course website.

Course website. The course website is www.tselilschramm.org/random-processes/stats221-winter22.html. There you will find an updated schedule, additional relevant readings, and posted homework assignments and solutions.

Canvas. We will use Canvas to post Zoom links for remote lectures (when necessary).

Gradescope. Homeworks will be turned in via Gradescope.

5 Coursework & Evaluation

Homework (40%). There will be weekly problem sets. The problem sets will be announced weekly in class on Wednesday, and will be due on Gradescope before the start of class on the following Wednesday (11:30 am Pacific). Collaboration is encouraged, but you must independently write your own solutions. We will drop your lowest homework score when computing your grade.

Final Presentation (60%). Students will give a 25 minute presentation on material not covered in the lectures. You may form teams of 2-3, and choose a topic of your interest from a list to be provided later in the quarter. To receive full marks, you must attend all other students' presentations.

6 Policies

The Honor Code. It is expected that you and I will follow Stanford's Honor Code in all matters relating to this online course. You are encouraged to meet and exchange ideas with your classmates while studying and working on homework assignments, but you are individually responsible for your own work and for understanding the material. You are not permitted to copy or otherwise reference another student's homework or computer code.

Late Homework Policy. Late work will not be accepted; however, in calculating your grade, we will drop your lowest homework score.

Accommodations. We are happy to provide accommodations, understanding that they may be necessary for student success. Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations.

Course Privacy Statement. As noted in the University's policy on recording and broadcasting courses, students may not audio or video record class meetings without permission from the instructor (and guest speakers, when applicable).